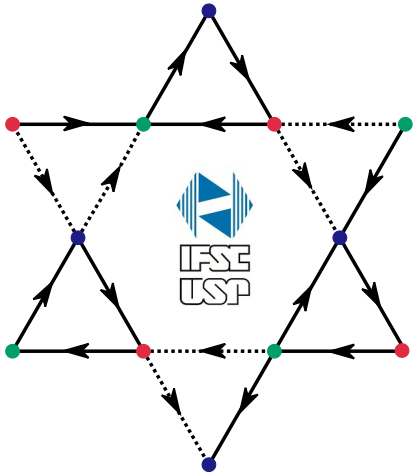
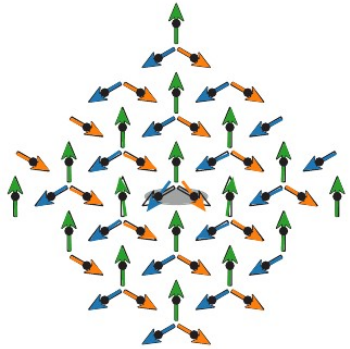
Noncoplanar magnetism and chiral spin liquids on the Kagome Lattice



João Augusto Sobral
University of São Paulo, IFSC

University of Innsbruck
Wednesday, 11/May/2022



Novel Quantum Materials
Prof. Eric C. Andrade's group (IFSC/USP)

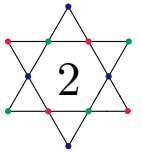
Main ref: (arXiv:2112.03327)

Acknowledgements



- Prof. Mathias S. Scheurer.
- Prof. Eric de Castro e Andrade.
- Prof. Rodrigo G. Pereira e Fabrizio G. Oliviero.
- Vitor Dantas Meireles.
- CAPES - Finance Code 001, via Grant No. 88887.474253/2020-00.

Outline

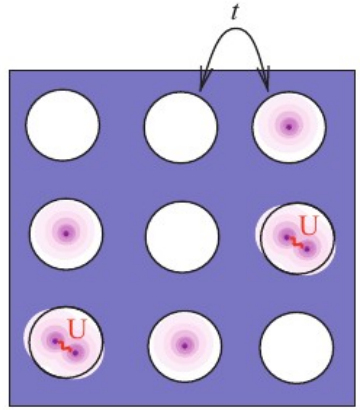


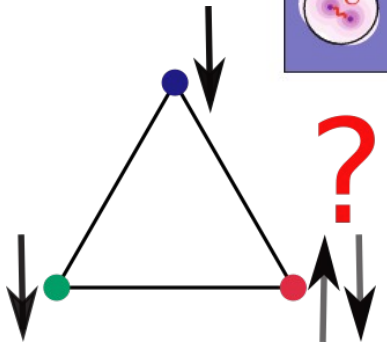
Mott Insulators, Frustrated Magnetism and Quantum Spin Liquids (QSL)

$$H = -t \sum_{j,\hat{a},\sigma} c^{\dagger}_{j+\hat{a}\sigma} c_{j\sigma} + \epsilon \sum_{j\sigma} c^{\dagger}_{j\sigma} c_{j\sigma} + U \sum_{j} n_{j\uparrow} n_{j\downarrow}$$

$$U/t \gg 1 \longrightarrow \mathcal{H} = J \sum_{<i,j>} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Th: P. Coleman, Introduction to many-body physics (2015)





- Absence of magnetic order up to $T \sim \mathcal{O}(mK)$;
- Fractionalized excitations, e.g.: Spinons, S=1/2;
- Gapless (emergent fermions coupled to gauge fields) and gapped (topological ground state degeneracy).

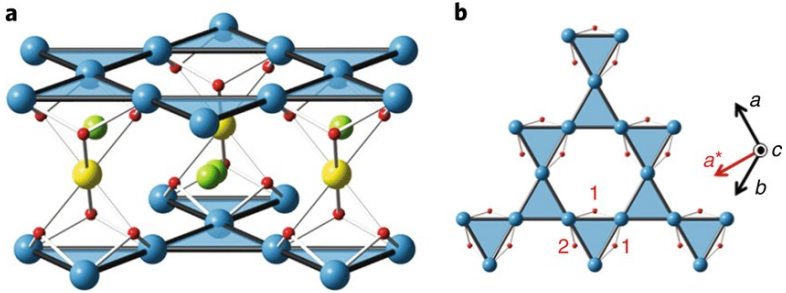
Th: Anderson, Mater. Res. Bull. **8**, 153 (1973) | Savary, Balents, Rep Prog. Phys. 2017 Anderson, Science **235**, 1196 (1987)

Frustrated magnetism on the Kagome lattice

Herbertsmithite

$ZnCu_3(OH)_6Cl_2$

Exp: P. Khuntia, *et al*, Nat. Phys. **16**, 469–474 (2020).

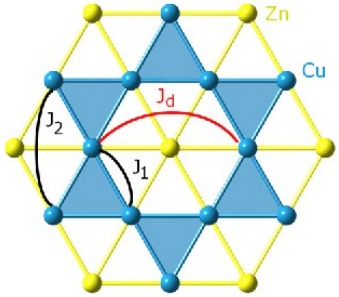


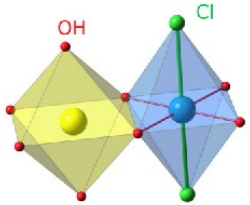
Kapellasite

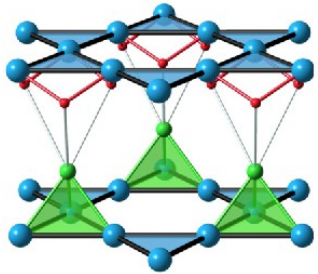
Exp: E. Kermarrec, *et al*, Phys. Rev. B **90**, 205103 (2014)

$Cu_3Zn(OH)_6Cl_2$

Kapellasite: a candidate for a GQSL?



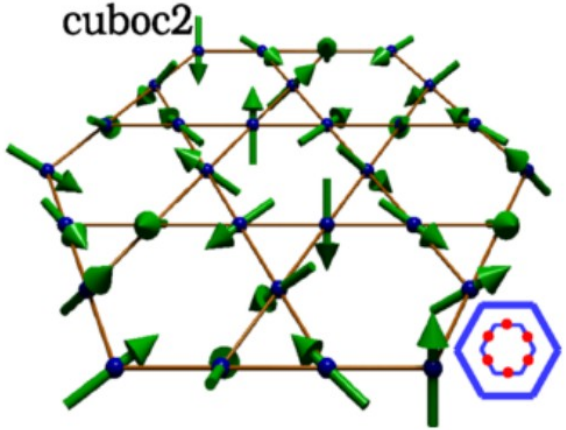




- Weak O-H-Cl coupling between Kagome planes.

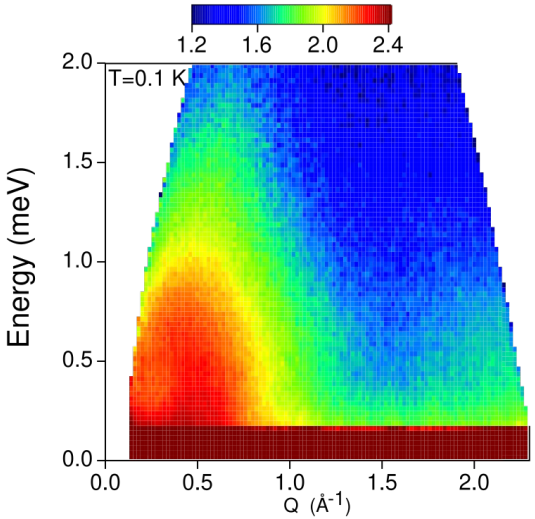
- Polymorphous to Herberthismithite (U(1) QSL e $J_1>0$);
- Gapless spin liquid ($\Delta \lesssim 10^{-3} J_1$);
- Short-range magnetic correlations of cuboc-2 type
- up to T=20 mK.

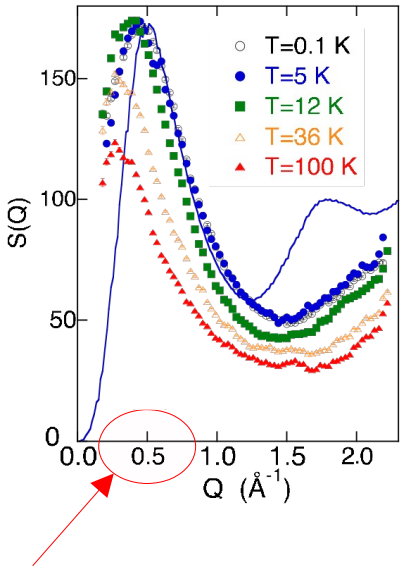
$$\mathbf{S}(\mathbf{r}_i) \cdot [\mathbf{S}(\mathbf{r}_j) \times \mathbf{S}(\mathbf{r}_k)]$$

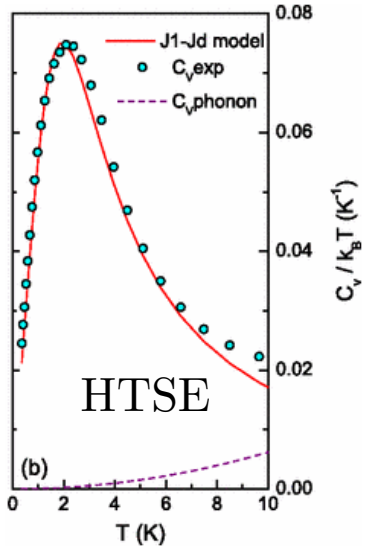


Exp: Fåk, Phys. Rev. Lett. 109, 037208T (2012)

Kapellasite: a candidate for a GQSL?







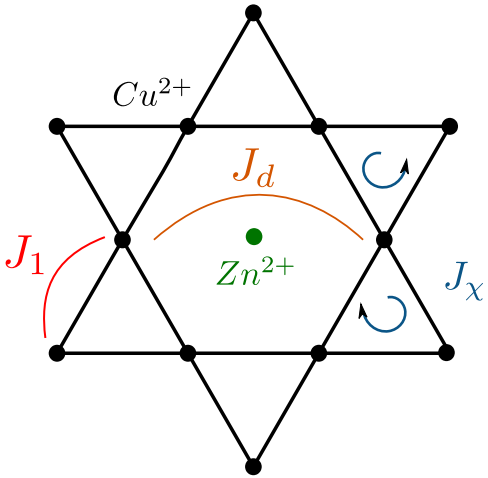
- Bragg peaks (cuboc-2);
- Continuum of excitations (spinons);
- Curie-Weiss Law ($\chi \propto 1/(T - \Theta_n)$, $\Theta_n = 9.5 \pm 1K$)

Exp: Fåk *et al*, Phys. Rev. Lett. 109, 037208T (2012)

Modified Heisenberg model $\mathrm{J}_1 - J_d - J_\chi$

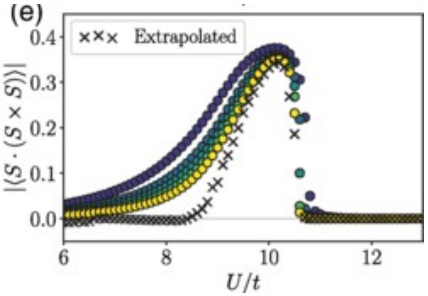
$$H = J_1 \sum_{\langle i,j \rangle} \mathbf{S}(\mathbf{r}_i) \cdot \mathbf{S}(\mathbf{r}_j) + J_d \sum_{ij\in} \mathbf{S}(\mathbf{r}_i) \cdot \mathbf{S}(\mathbf{r}_j) +$$

$$+ J_\chi \sum_{ijk\in\triangle} \mathbf{S}(\mathbf{r}_i) \cdot [\mathbf{S}(\mathbf{r}_j) \times \mathbf{S}(\mathbf{r}_k)] - J_\chi \sum_{ijk\in\triangledown} \mathbf{S}(\mathbf{r}_i) \cdot [\mathbf{S}(\mathbf{r}_j) \times \mathbf{S}(\mathbf{r}_k)],$$



$\mathrm{J}_1 < 0$ (FM), $J_d > 0$ (AFM) e $J_\chi > 0$

- Staggered chirality;
- Noncoplanar phases and CSLs;
- Amplified near the Mott transition;
- Floquet engineering.



Th: Claassen *et al* , Nat. Com. v.8, 1192 (2017) | Quito *et al*, Phys. Rev. Let. 126, 177201 (2021) | Cookmeyer *et al* Phys. Rev. Lett. 127, 087201 (2021) | Szasz *et al* Phys. Rev. X 10, 021042 (2020)

Classical Phases: Gradient descent and SSSF ($\boldsymbol{S}$ classical)

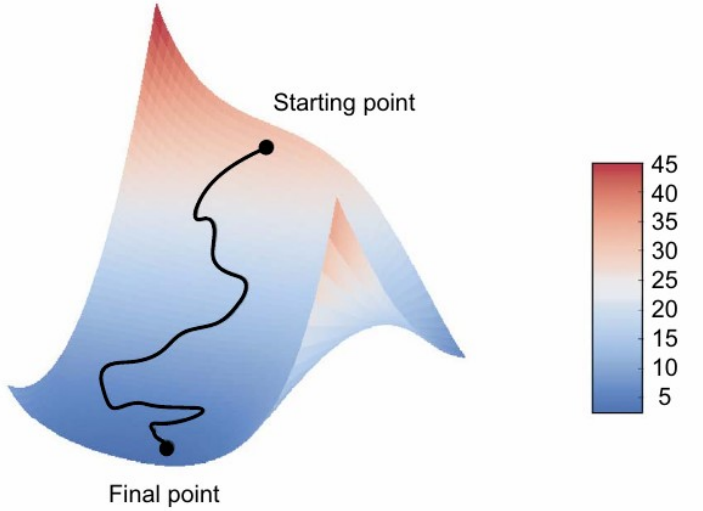
$$\mathcal{H}_{GD} = \sum_i \boldsymbol{S}_i \cdot \boldsymbol{h}_i$$

$$\boldsymbol{h}_i = J_1 \sum_j \boldsymbol{S}_j + J_d \sum_{j\in} \boldsymbol{S}_j + J_\chi \sum_{j,k\in\triangle\triangledown} (\boldsymbol{S}_j \times \boldsymbol{S}_k)$$

$$-\nabla_i \mathcal{H}_{GD}(\boldsymbol{S}_i) \parallel \boldsymbol{h}_i$$

$$\nabla_i \mathcal{H}_{GD} = \frac{\partial \mathcal{H}_{GD}}{\partial S_i^x}\hat{x} + \frac{\partial \mathcal{H}_{GD}}{\partial S_i^y}\hat{y} + \frac{\partial \mathcal{H}_{GD}}{\partial S_i^z}\hat{z} = \boldsymbol{h}_i$$

$$\boldsymbol{S}_{i+1} = (1-\gamma)\boldsymbol{S}_i - \gamma \nabla_i \mathcal{H}(\boldsymbol{S}_i)$$



Chollet, Deep Learning with Python (2021)

- Accesible experimentally through inelastic neutron scattering data (INS).

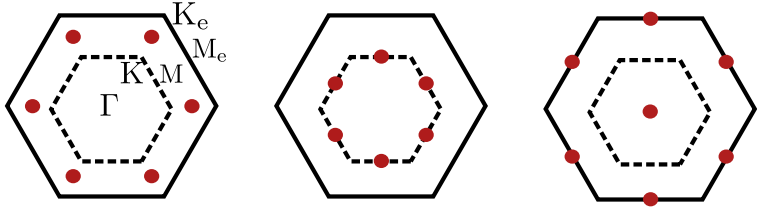
SSSF

$$\boldsymbol{S}_{\boldsymbol{k}} = \frac{1}{\sqrt{N}} \sum_{\boldsymbol{r}} e^{-i\boldsymbol{r}\cdot\boldsymbol{k}} \boldsymbol{S}_{\boldsymbol{r}}$$

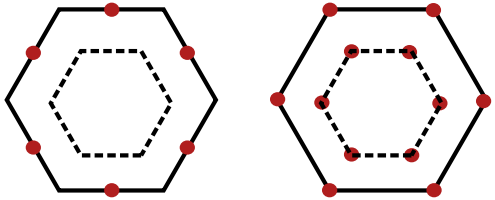
$$\mathcal{S}(m_1, m_2) = \boldsymbol{S}_{\boldsymbol{k}} \cdot \boldsymbol{S}_{-\boldsymbol{k}} = |\boldsymbol{S}_{\boldsymbol{k}}|^2$$

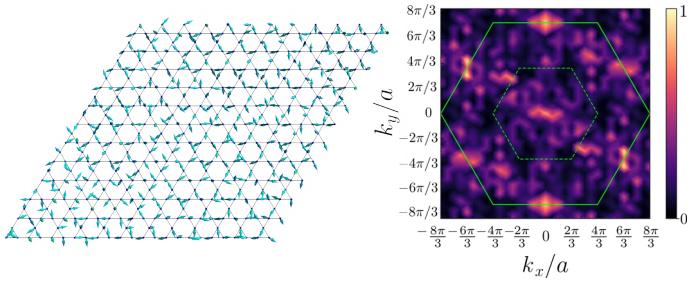
Classical Phases: Gradient descent and SSSF ($\boldsymbol{S}$ classical)

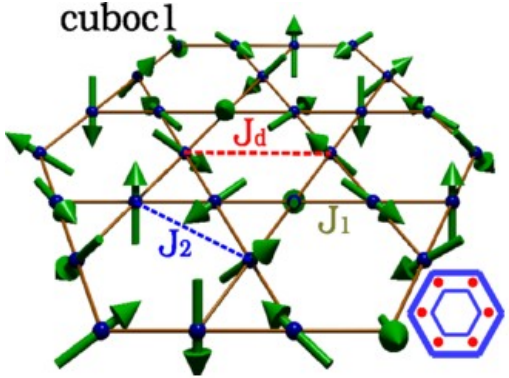
cuboc-1 cuboc-2 FM

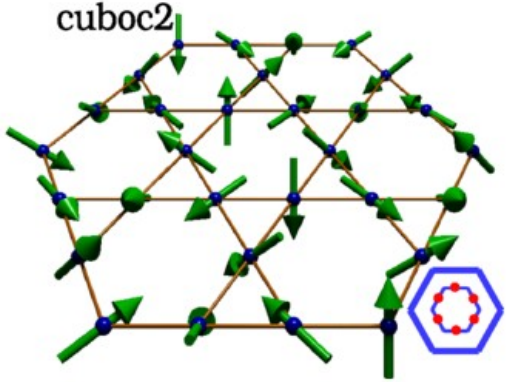


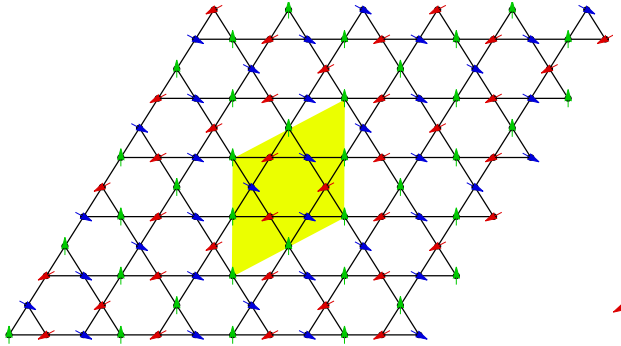
$\boldsymbol{Q} = 0 \times 0$ $\boldsymbol{Q} = \sqrt{3} \times \sqrt{3}$



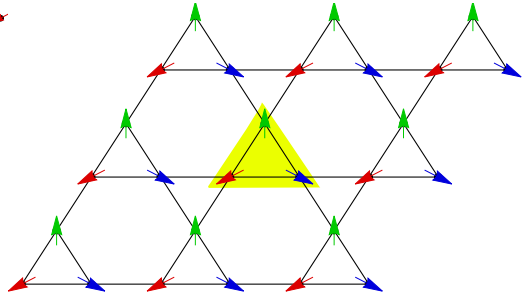








$\boldsymbol{Q} = \sqrt{3} \times \sqrt{3}$



$\boldsymbol{Q} = 0 \times 0$

Parton Construction and Gutzwiller Projection ($\mathbf{S} = 1/2$)

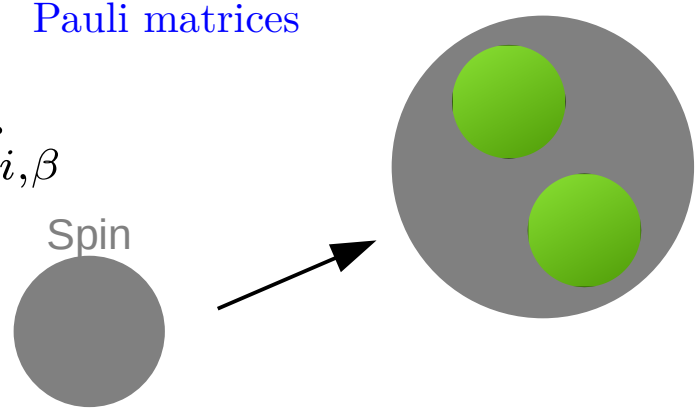
Abrikosov Fermions

$$S_i^{\gamma} = \frac{1}{2} \sum_{\alpha,\beta} f_{i,\alpha}^{\dagger} \sigma_{\alpha\beta}^{\gamma} f_{i,\beta}$$

Pauli matrices

Complex fermions

Spin



$$\hat{S}_i^{+} = f_{i\uparrow}^{\dagger} f_{i\downarrow}, \hat{S}_i^{-} = f_{i\downarrow}^{\dagger} f_{i\uparrow} \quad e \quad \hat{S}_i^{z} = \frac{1}{2}\left(n_{i\uparrow} - n_{i\downarrow}\right)$$

Single-occupancy constraint (local)

$$n_{i\uparrow} + n_{i\downarrow} = 1$$

MFT decoupling

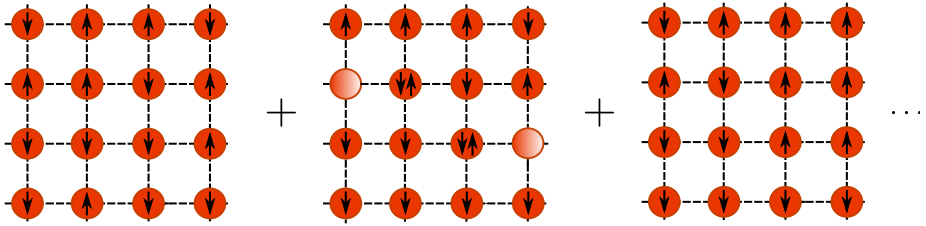
$$\chi_{ij} = f_{i\uparrow}^{\dagger} f_{j\uparrow} + f_{i\downarrow}^{\dagger} f_{j\downarrow}$$

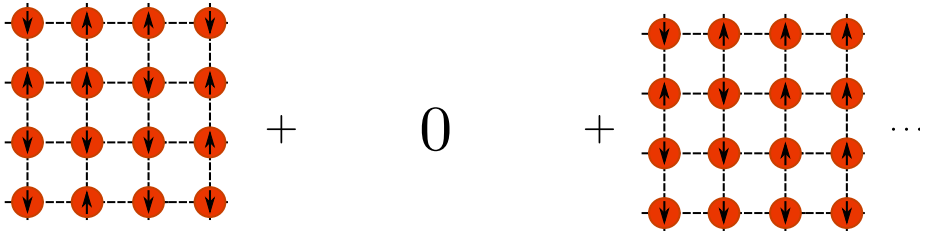
$$\boxed{H_{MF}\Psi_{MF} = E\Psi_{MF}}$$

Parton Construction and Gutzwiller Projection

- Ansatz described by a Slater determinant for a singlet state (S_{tot}=0), with spin ½ excitations.
- The occupancy can be imposed **exactly** through Variational Monte Carlo or MPS, by using the **Gutzwiller projection**:

$$|\psi\rangle = \hat{P}_G\,|\psi_{MF}\rangle \qquad \hat{P}_G = \prod_i (2 - n_i)\, n_i \qquad n_i = \sum_\alpha f^\dagger_{i,\alpha} f_{i,\alpha}$$

$|\psi_{Slater}\rangle =$  $+ \cdots$

$\hat{P}_G\,|\psi_{Slater}\rangle =$  $+ \quad 0 \quad + \cdots$

Gros, Ann. Phys. **189**, 53 (1989) / Zou, Y., et al Rev. Mod. Phys. **89**, 025003 (2017). / Gutzwiller, M., Phys. Rev. A **137**, A1726 (1964)

Parton construction and VMC

Configurations

$$\langle \mathcal{O} \rangle = \frac{\langle \psi | \mathcal{O} | \psi \rangle}{\langle \psi | \psi \rangle} = \sum_{\alpha\beta} \langle \alpha | \mathcal{O} | \beta \rangle \frac{\langle \psi | \alpha \rangle \langle \beta | \psi \rangle}{\langle \psi | \psi \rangle}.$$

$$\langle \mathcal{O} \rangle = \sum_{\alpha} \left(\sum_{\beta} \langle \alpha | \mathcal{O} | \beta \rangle \frac{\langle \beta | \psi \rangle}{\langle \alpha | \psi \rangle} \right) \frac{|\langle \alpha | \psi \rangle|^2}{\langle \psi | \psi \rangle} = \sum_{\alpha} f(\alpha)\, p(\alpha)$$

$$\mathrm{f}(\alpha) = \left(\sum_{\beta} \langle \alpha | \mathcal{O} | \beta \rangle \frac{\langle \beta | \psi \rangle}{\langle \alpha | \psi \rangle} \right)$$

Statistical average of local operator

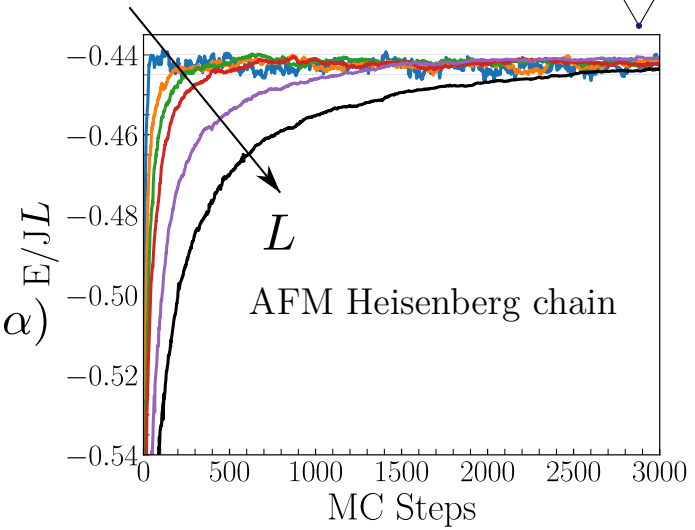
$$\mathrm{p}(\alpha) = \frac{|\langle \alpha | \psi \rangle|^2}{\langle \psi | \psi \rangle} \geq 0$$

$$\sum_{\alpha} p(\alpha) = 1.$$

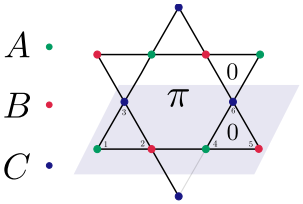
Probability distribution

$$\mathrm{T}(\alpha \to \alpha') = \min\left[1, \frac{p(\alpha')}{p(\alpha)}\right] = \begin{cases} \frac{p(\alpha')}{p(\alpha)} & , \frac{p(\alpha')}{p(\alpha)} < 1 \\ 1 & , \frac{p(\alpha')}{p(\alpha)} > 1 \end{cases},$$

Markov Chain - > Equilibrium



QSLs on the Kagome lattice (Heis. model J_1)



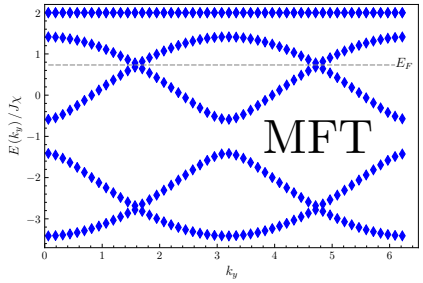
$$\begin{cases} \phi_{\hexagon} = \sum_{i,j\in\hexagon} \chi_{ij} = -1 \\ \phi_{\triangle} = \sum_{i,j\in\triangle} \chi_{ij} = 1 \end{cases}$$

$$\chi_{i,j} \to \bar{\chi}_{i,j} e^{-i a_{i,j}}$$

$$H = J_1 \sum_{\langle i,j \rangle} \mathbf{S}(\mathbf{r}_i) \cdot \mathbf{S}(\mathbf{r}_j)$$

- CSL
- Uniform RVB with spinon Fermi surface.
- U(1) Dirac QSL (Stable)?

Spin liquid	$8 \times 8 \times 3$ lattice	$12 \times 12 \times 3$ lattice
SL-$[\frac{\pi}{2}, 0]$	$-0.4010(1)$	$-0.4010(1)$
SL-$[\pm\frac{\pi}{2}, 0]$	$-0.3907(1)$	$-0.3910(1)$
SL-$[\frac{\pi}{2}, \pi]$	$-0.3814(1)$	$-0.3822(1)$
SL-$[0, 0]$	$-0.4115(1)$	$-0.4121(1)$
SL-$[0, \pi]$	$-0.42866(2)$	$-0.42863(2)$



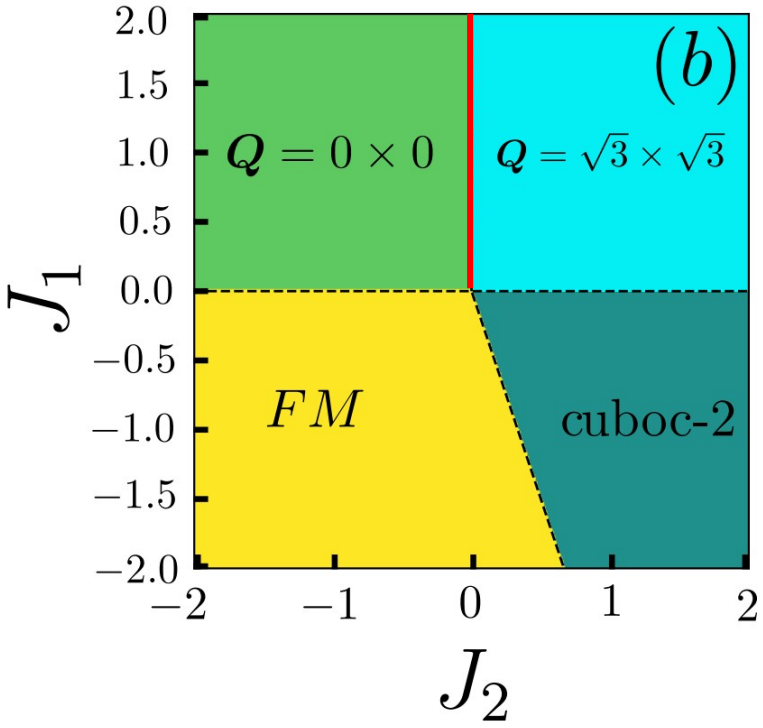
Method	Energy per site
Exact diagonalization [11]	-0.43
Coupled cluster method [14]	-0.4252
Spin-wave variational method [15]	-0.419

Herbertsmithite

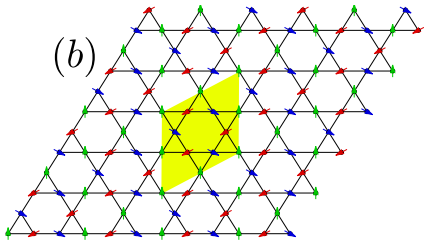
M. B. Hastings, Phys. Rev. B 63, 014413 (2000) | Y. Ran, M. Hermele, P. A. Lee, and X.-G. Wen, Phys. Rev. Lett. 98, 117205 (2007). | Y.-C. He, M. P. Zaletel, M. Oshikawa, and F. Pollmann, Phys. Rev. X 7, 031020 (2017)

QSLs on the Kagome lattice (Heis. model J_1-J_2)

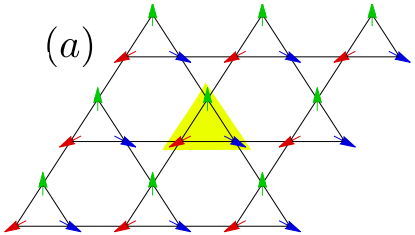
$S \to \infty$



$$H = J_1 \sum_{\langle i,j \rangle} \mathbf{S}(\mathbf{r}_i) \cdot \mathbf{S}(\mathbf{r}_j) + J_2 \sum_{\langle\langle i,j \rangle\rangle} \mathbf{S}(\mathbf{r}_i) \cdot \mathbf{S}(\mathbf{r}_j)$$

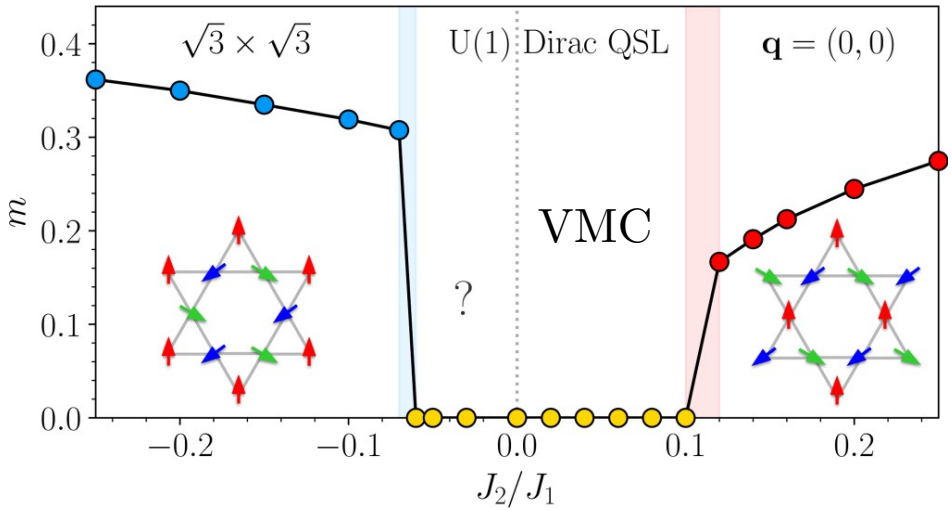


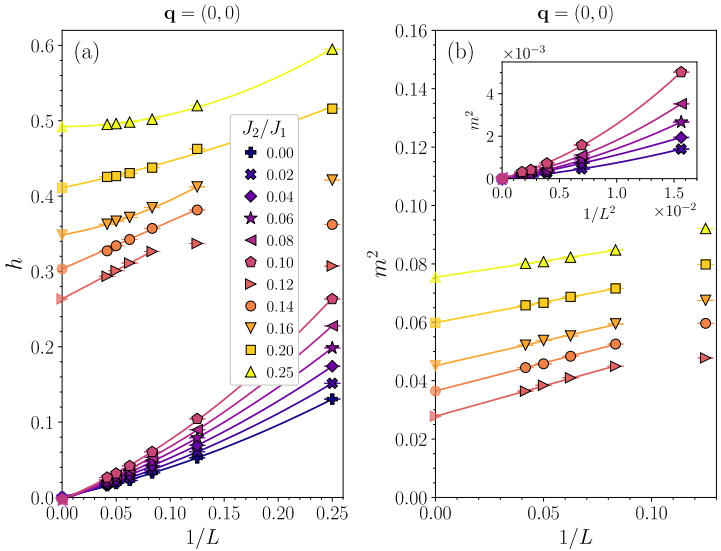
$Q = \sqrt{3} \times \sqrt{3}$



$Q = 0 \times 0$

QSLs on the Kagome lattice (Heis. model J_1-J_2)





$$\mathrm{H}_{MFT} = \sum_{\langle i,j \rangle, \alpha} \chi_{ij} f^{\dagger}_{i\alpha} f_{j\alpha} + h \sum_i \boldsymbol{m}_i \cdot \boldsymbol{S}_i$$

$$m^2 = \lim_{|i-j| \to \infty} \langle \boldsymbol{S}_i \cdot \boldsymbol{S}_j \rangle$$

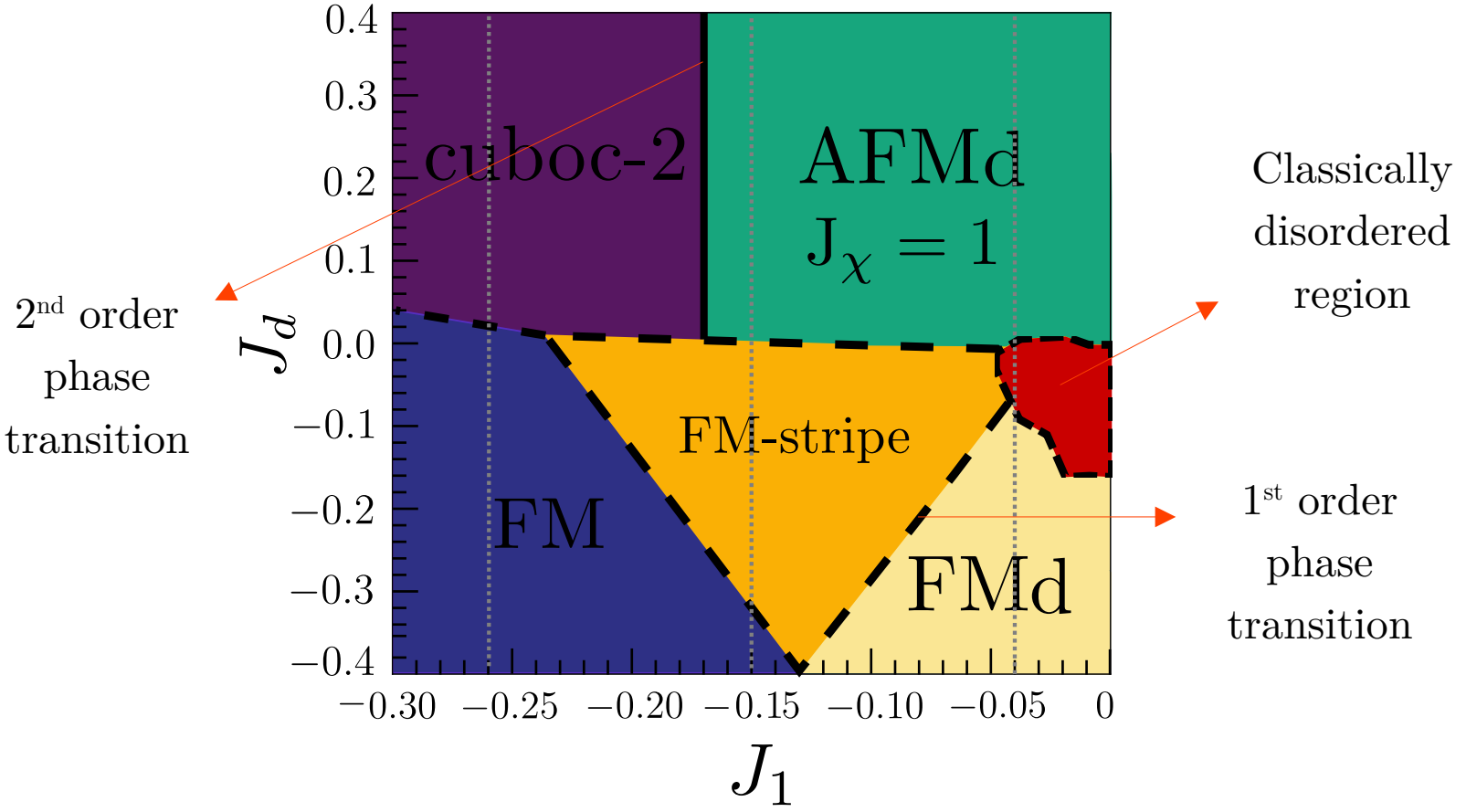
SDW

Francesco Ferrari, et al. Phys. Rev. B 104, 144406 (2021) | Iaconis, SciPost Phys. 4, 003 (2018)

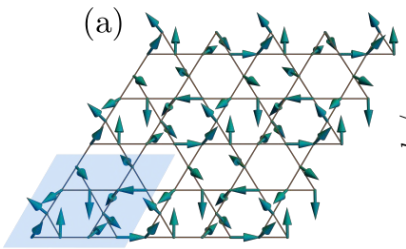
1 Noncoplanar ordered phases on the Kagome lattice

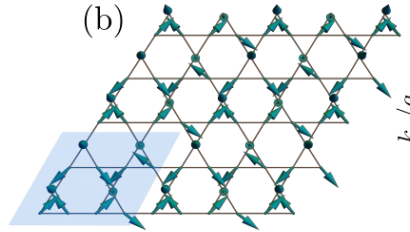
2 Gapless chiral spin liquid on the Kagome lattice

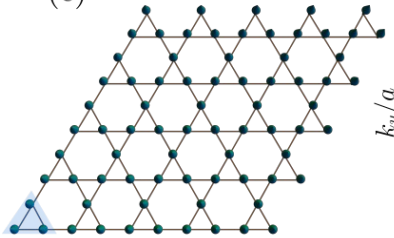
Classical phase diagram for the $J_1 - J_d - J_\chi$ model

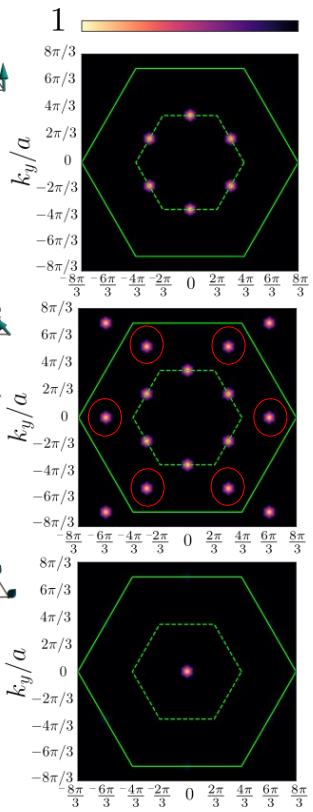


Static spin structure factor (SSSF) and ordered phases

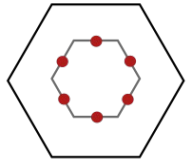




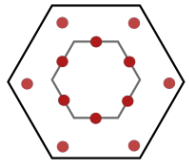




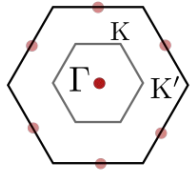
cuboc-2



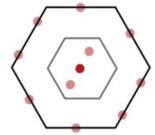
AFMd

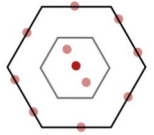


FM

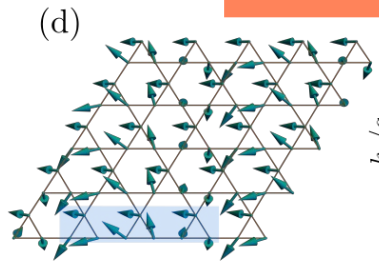


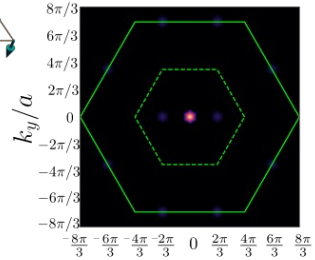
C_3 lattice symmetry

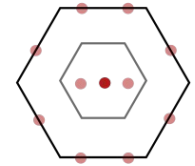


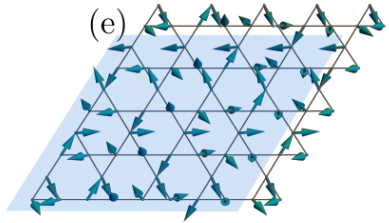


FM-stripe

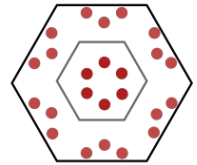


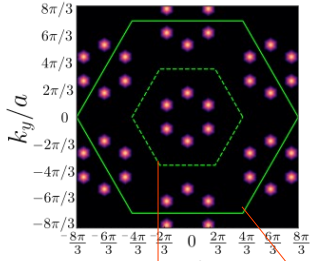






FMd

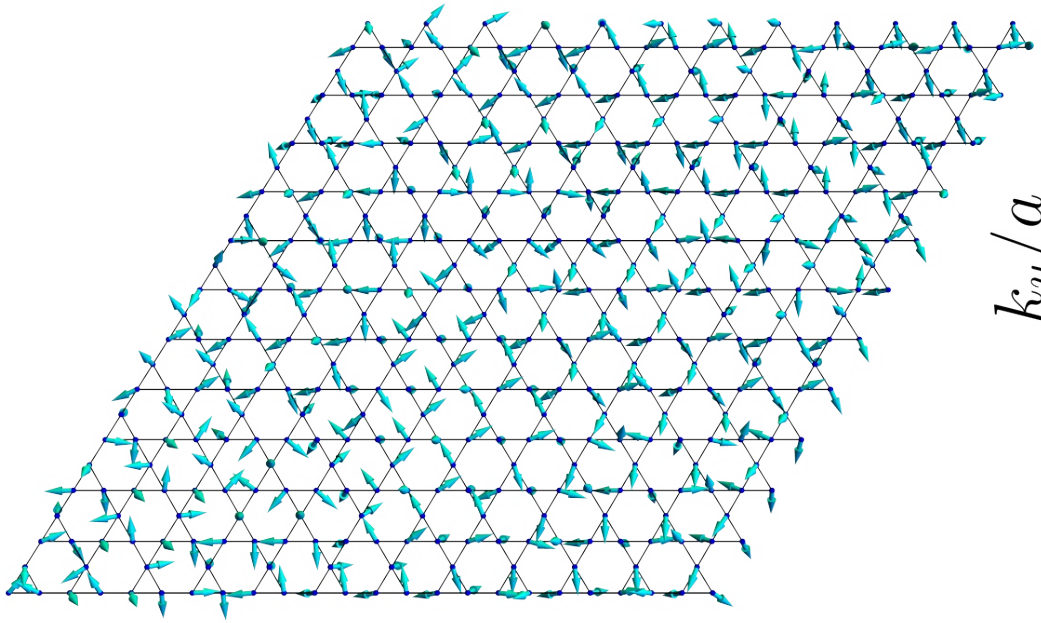


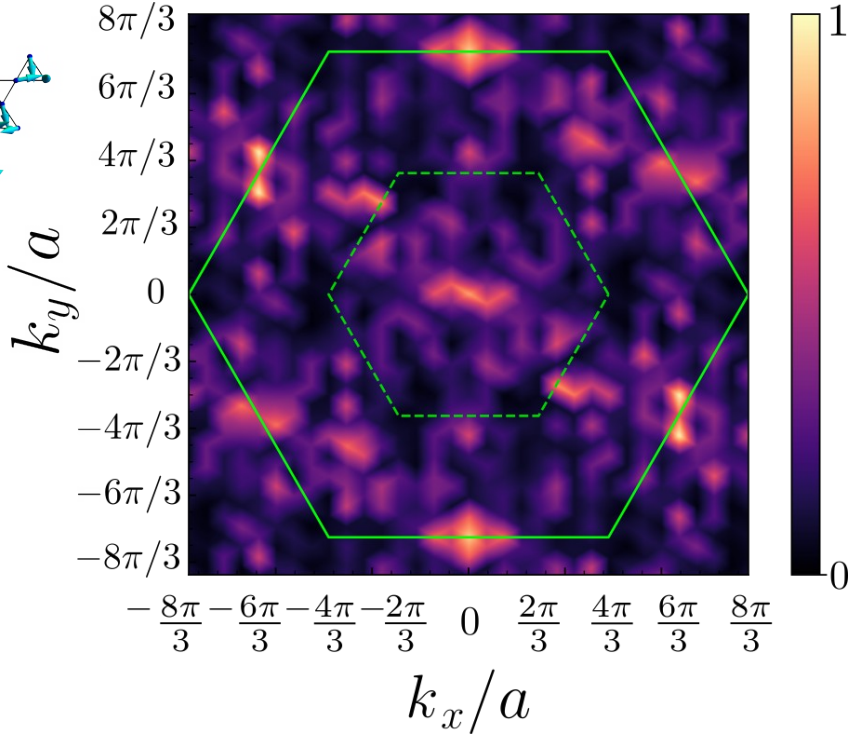


EBZ

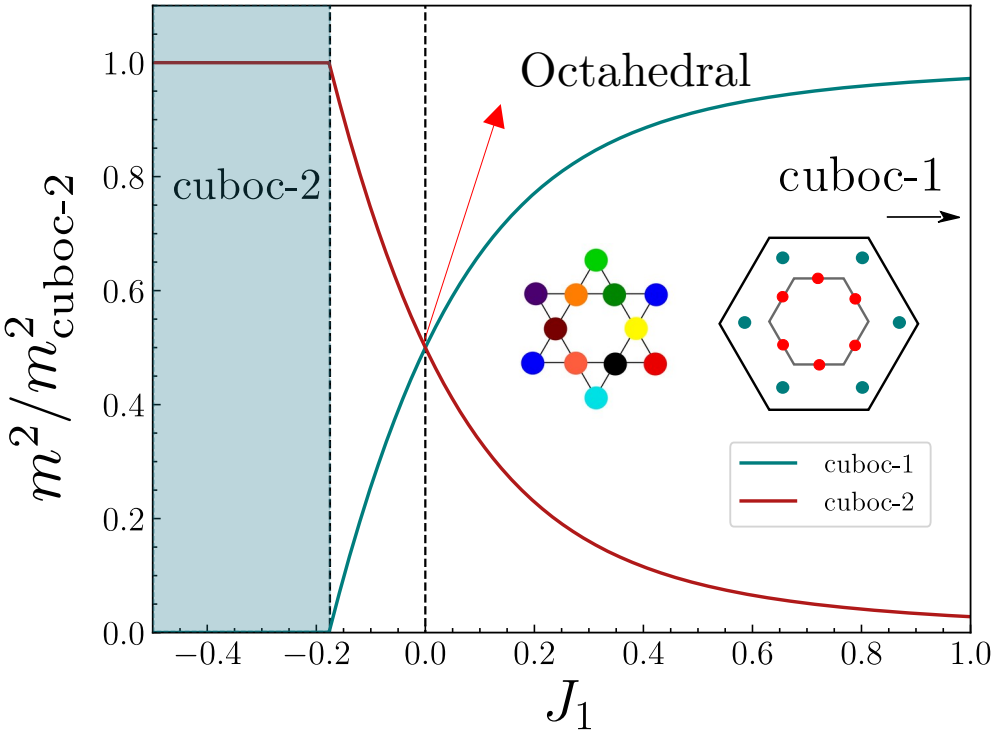
1st BZ

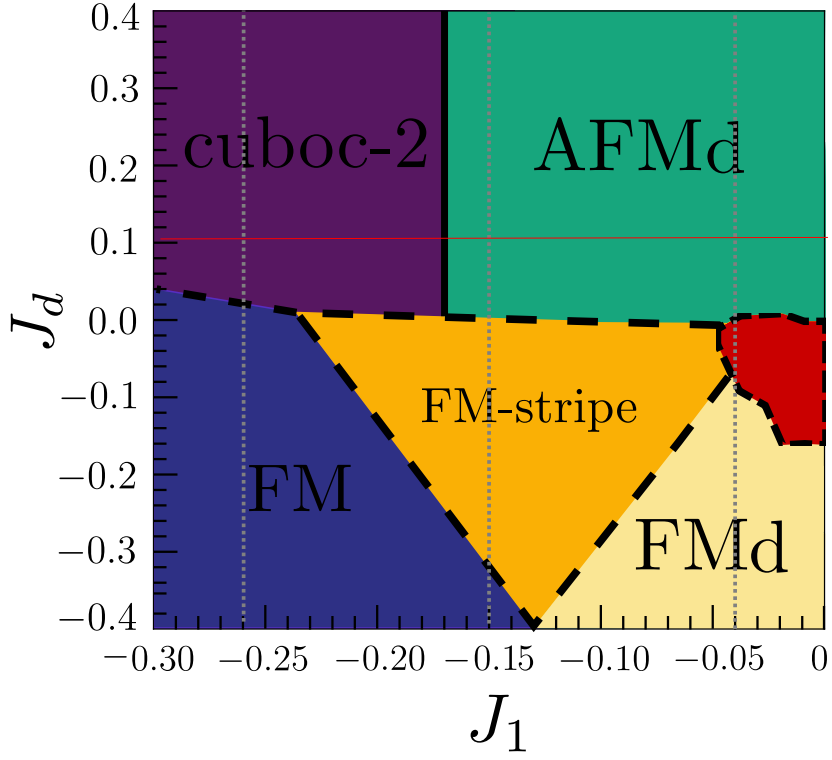
Classically disordered region



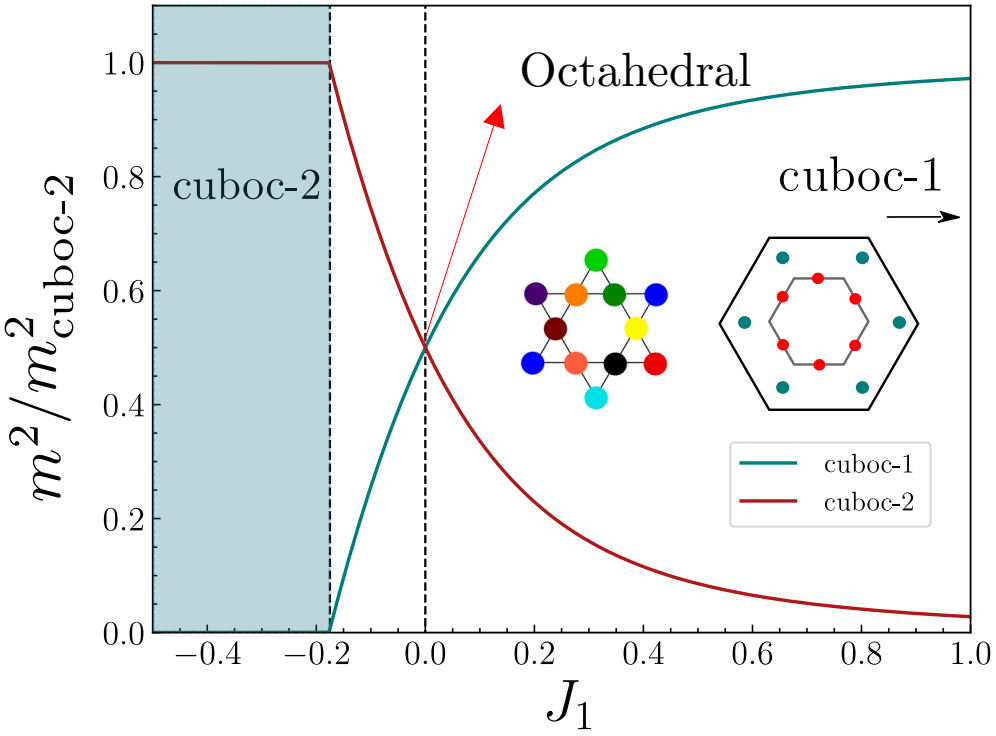


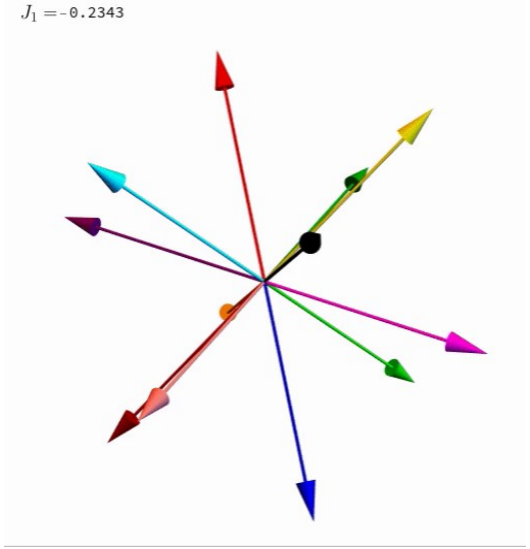
Cuboc-2 and cuboc-1 phase transition (AFMd)





Cuboc-2 and cuboc-1 phase transition (AFMd)

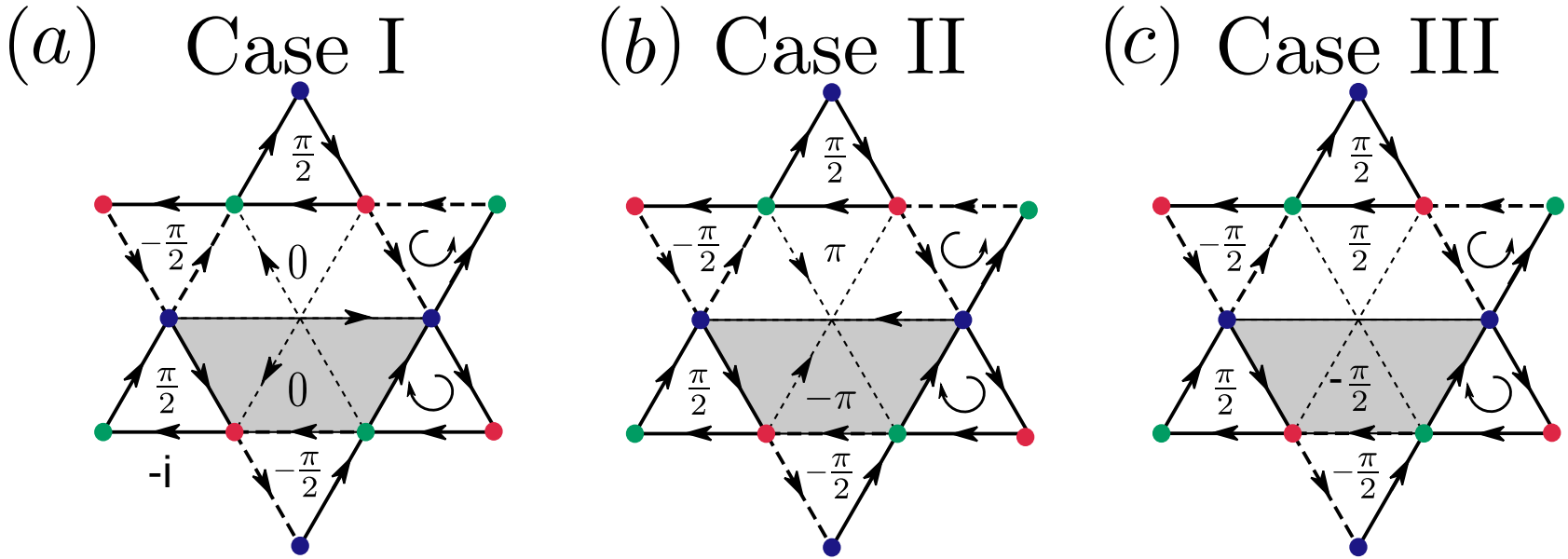




1 Noncoplanar ordered phases on the kagome lattice

2 Gapless chiral spin liquid on the kagome lattice

Parton mean-field theory: ansatz $[\pm\pi/2, 0]$



Bieri *et al*, Phys. Rev. B 93, 094437 (2016) | Pereira and Bieri, SciPost Phys. 4, 004 (2018) | Bauer et al, Phys. Rev. B 99, 035155 (2019)

Parton mean-field theory

$$S_i^\gamma = \frac{1}{2}\sum_{\alpha,\beta} f_{i,\alpha}^\dagger \sigma_{\alpha\beta}^\gamma f_{i,\beta}$$

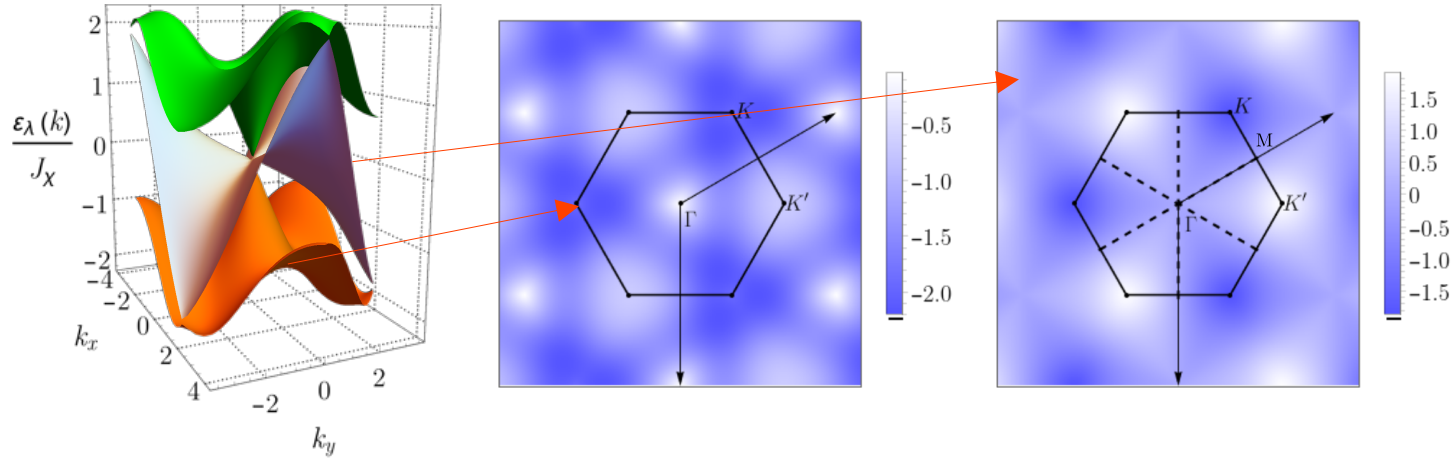
MFT parameters

$$\kappa_1 = \frac{3}{8} J_\chi \chi_1^2 - J_1\chi_1, \quad \kappa_d = J_d\chi_d$$

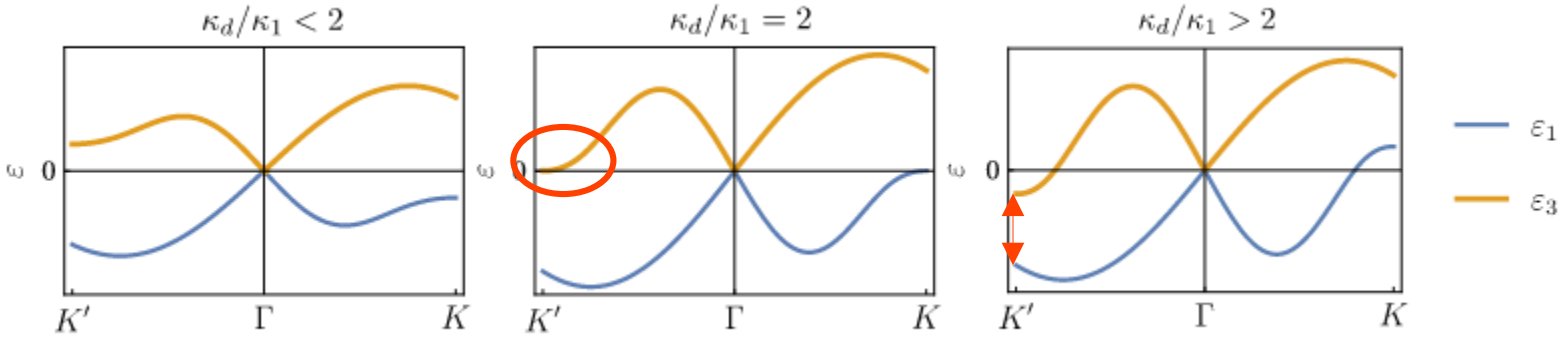
- Energetic favoring for Case II;
- Nodal lines (dashed lines);
- Dirac Cones in Γ.

Majorana, J_x only

Bauer et al, Phys. Rev. B 99, 035155 (2019)



Parton mean-field theory (Instabilities)

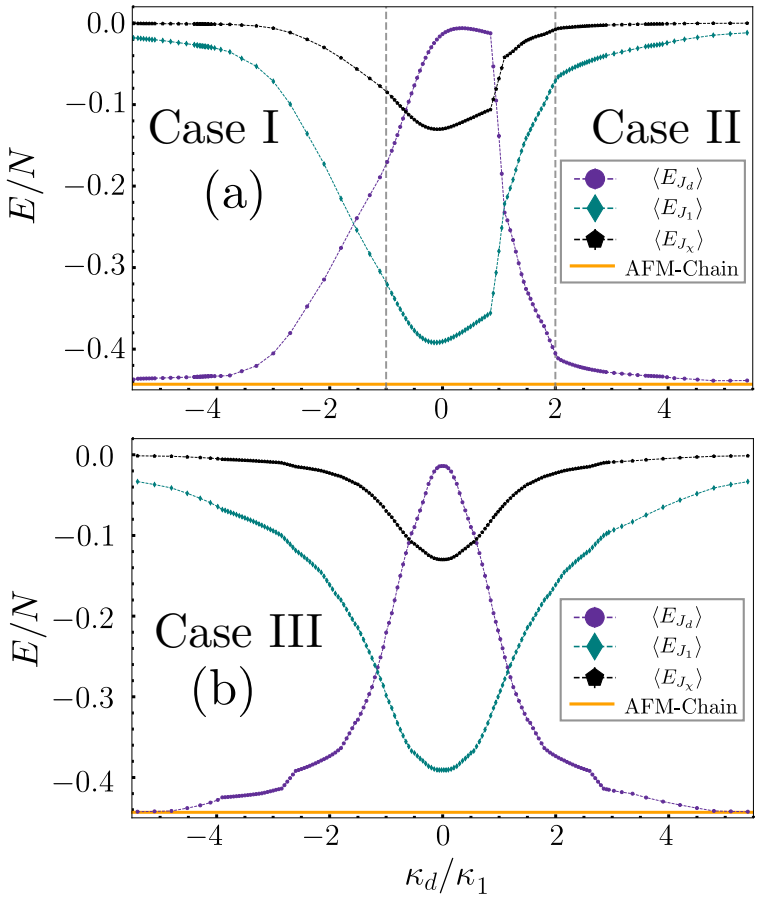


- Nesting instability of the U(1) CSL to ordered phases (Fermi pockets in $\kappa_d/\kappa_1 > 2$ and E_F crossing);
- Flattening of central band in $\kappa_d/\kappa_1 \approx -1$ (Case I).

Krüger and Janssen, Phys. Rev. B 104, 165133 (2021)

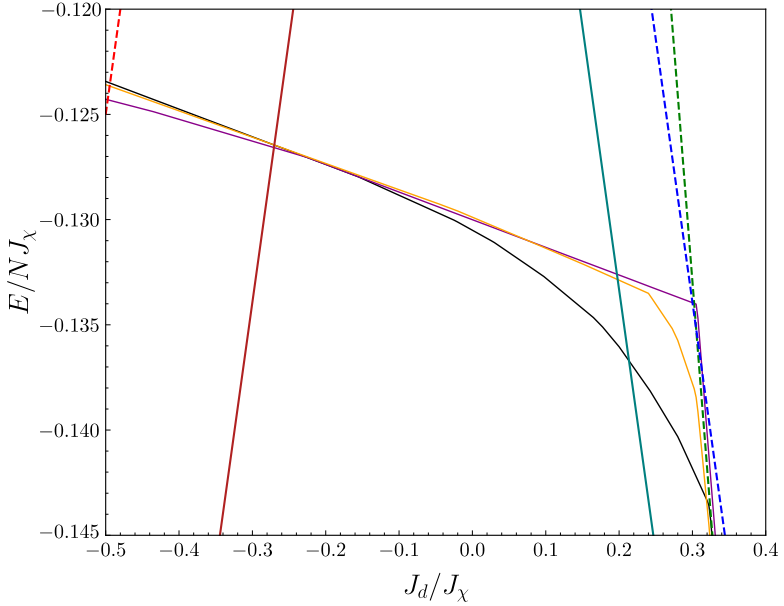
$$\kappa_d/\kappa_1 \in [-1, 2]$$

Ground state energy (VMC)



$$\mathrm{E}_{AFM} = J_d \left(\frac{1}{4} - \ln(2) \right)$$

$$F(\kappa_d/\kappa_1) = J_1 \langle E_{J_1} \rangle + J_d \langle E_{J_d} \rangle + J_\chi \langle E_{J_\chi} \rangle$$

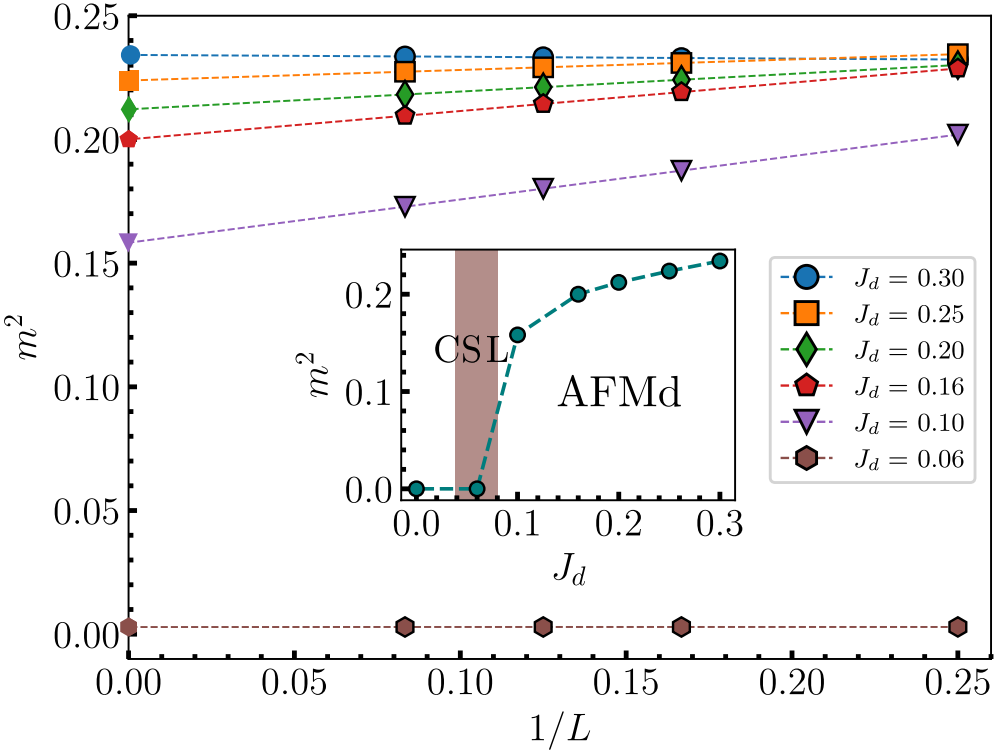


$$\mathrm{J}_1 = 0$$

Case I
Case II
Case III
1/2 - Chain (AFM)
cuboc-2
AFMd
FMd
FM

Magnetization and Instabilities

$(J_1 = -0.01, J_\chi = 1.00)$

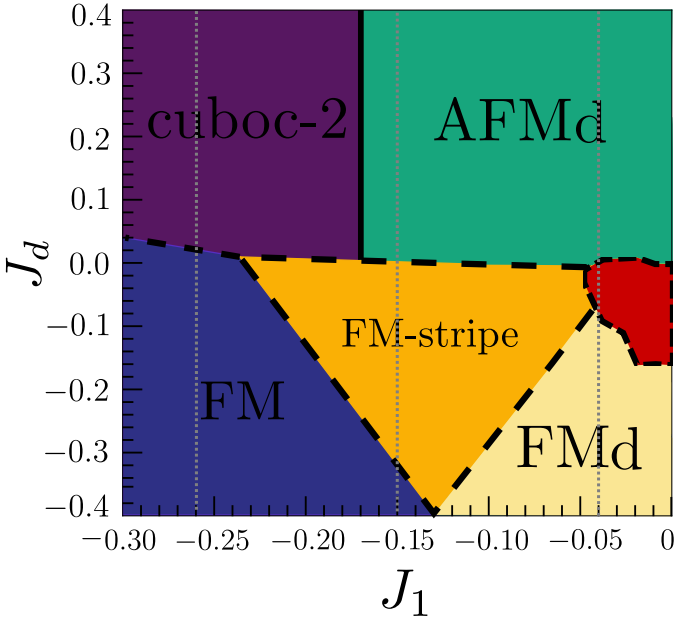


$$\mathrm{H}_{MFT} = \sum_{\langle i,j\rangle,\alpha} \chi_{ij} f^{\dagger}_{i\alpha} f_{j\alpha} + \\ + \mathrm{h} \sum_i \boldsymbol{m}_i \cdot \boldsymbol{S}_i$$

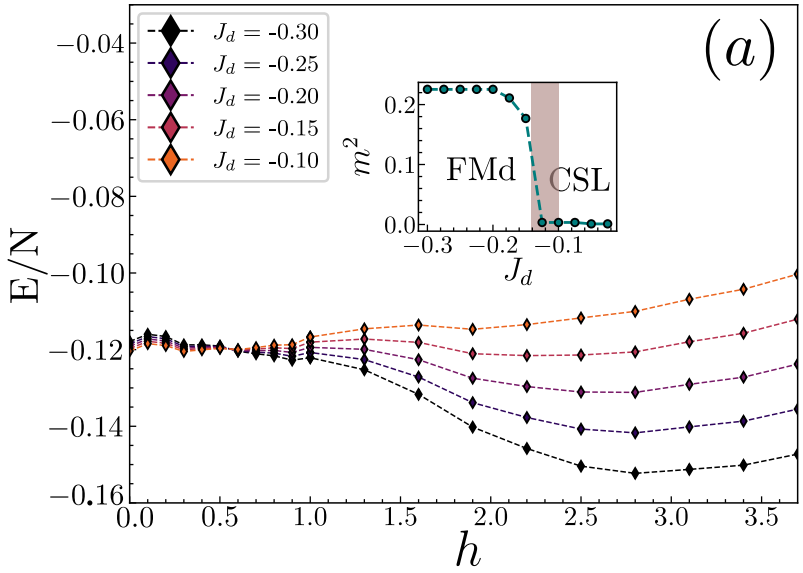
$$m^2 = \lim_{|i-j|\to\infty} \langle \boldsymbol{S}_i \cdot \boldsymbol{S}_j \rangle$$

- SDW ansatz;
- Finite-size scaling.

Transition between CSL and ordered phases

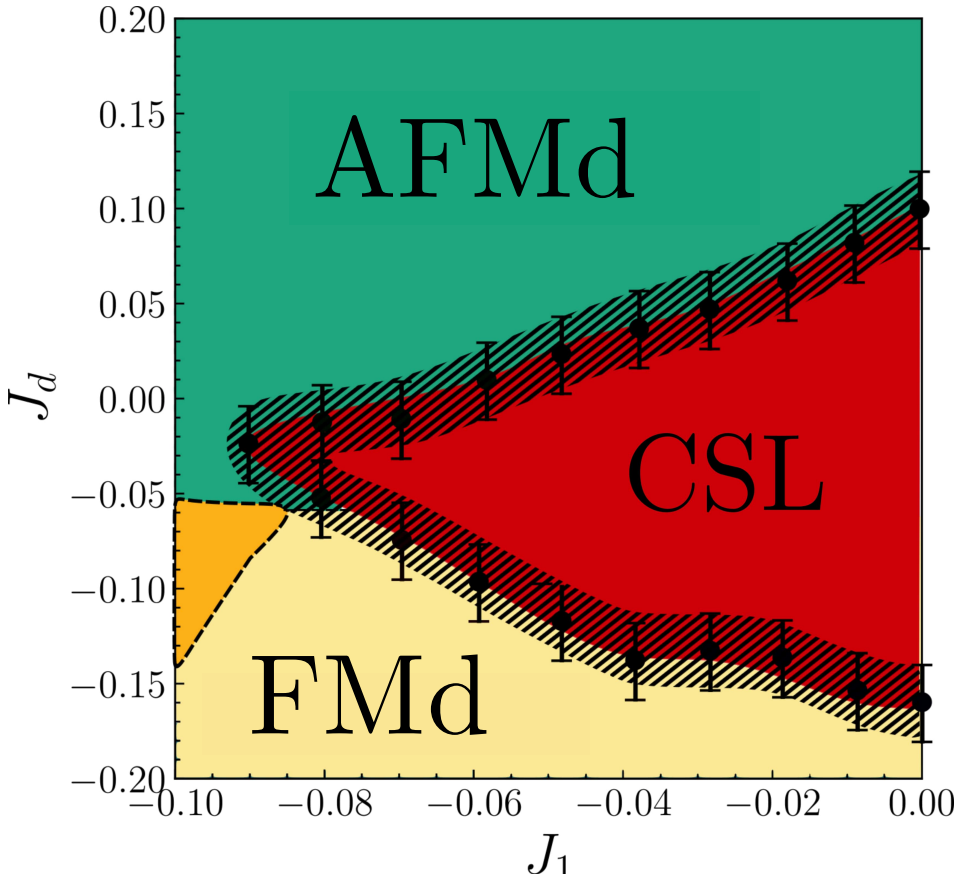


$(J_1 = -0.02)$



- Phases with h≠ 0 are adiabatically connected to the ordered phases.

VMC results near the CSL



Conclusions and outlook

- New ordered phases: FMd, AFMd, FM-stripe;
- Continuous phase transition between phases cuboc-1, cuboc-2, octahedral e AFMd;
- Classically disordered region extended -> Competitive CSL in the phase diagram;

Main ref: (arXiv:2112.03327)

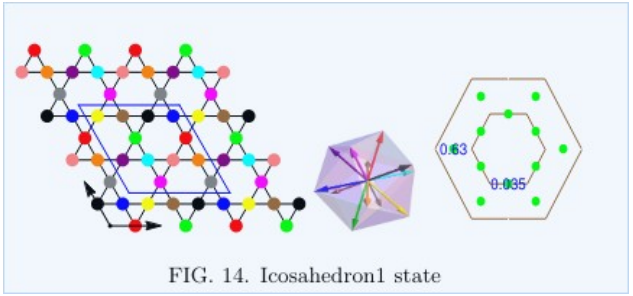
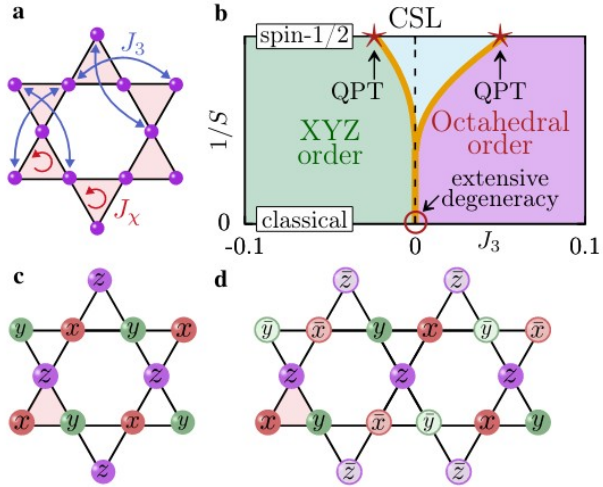


FIG. 14. Icosahedron1 state

- AFMd and connection to other ordered phases?
  Mondal et al, Jour. of Phys.: Cond. Mat. 33 (50), 505801 (2021)
- Thermal evolution of ordered phases AFMd, FMd and FM-stripe (Order by disorder)?
  Pitts et al, arXiv:2110.11427 (2021)
- Nature of quantum phase transitions to the GCSL?
  Wietek and Läuchli, Phys. Rev. B 95, 035141 (2016) | Hickey et al, Phys. Rev. B 96, 115115 (2017) | Bose et al, arXiv:2204.10329 (2022)



Acknowledgements

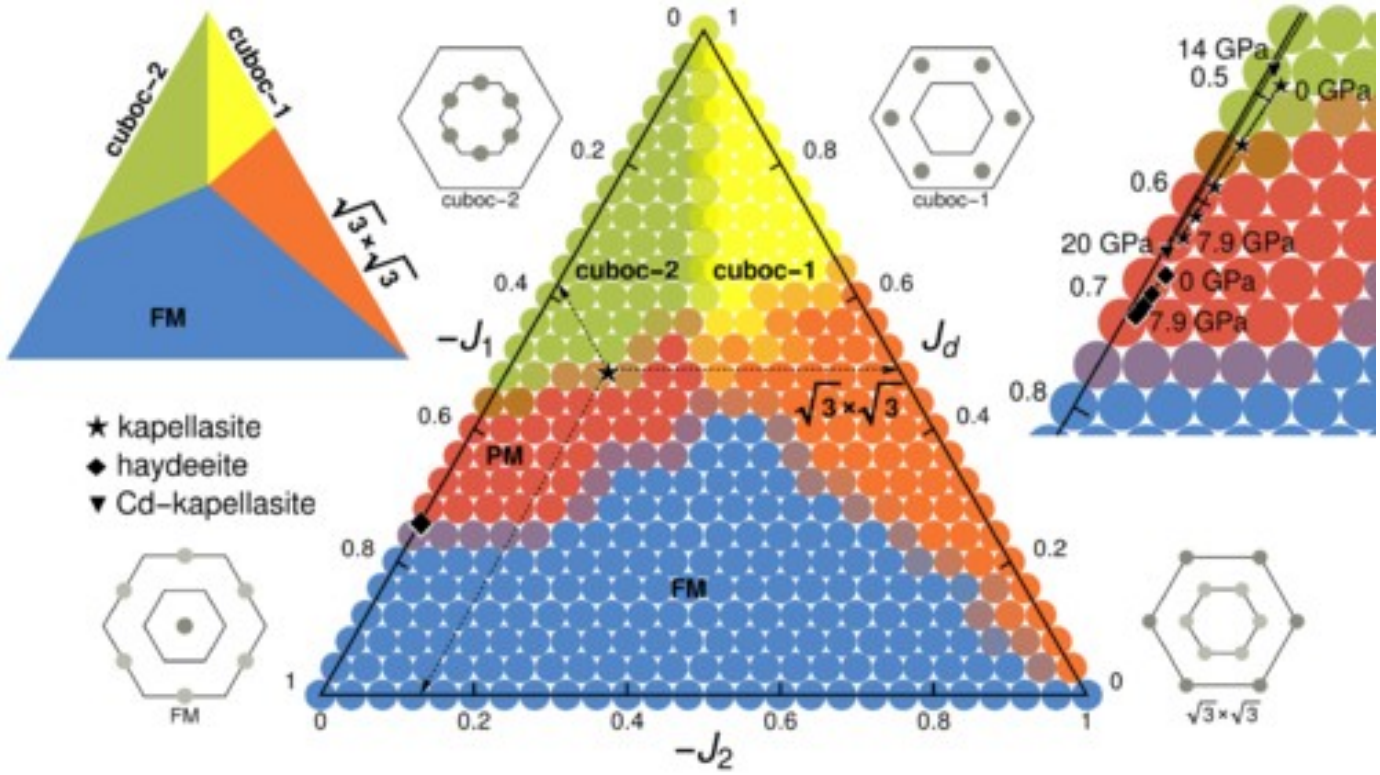


- Prof. Mathias S. Scheurer.
- Prof. Eric de Castro e Andrade.
- Prof. Rodrigo G. Pereira e Fabrizio G. Oliviero.
- Vitor Dantas Meireles.
- CAPES - Finance Code 001, via Grant No. 88887.474253/2020-00.

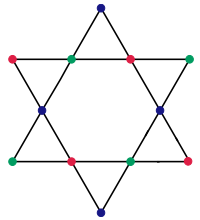
CAPES

1 Extra slides

Quantum phase diagram of the J_1-J_2 Heis. model

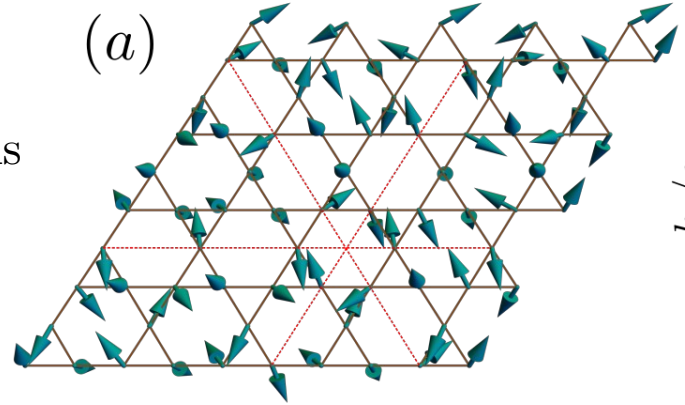


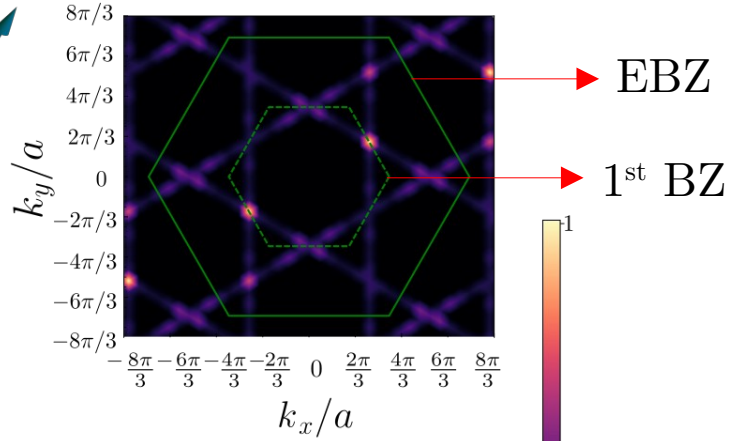
Iqbal *et al*, *Phys. Rev. B 92, 220404(2015)*



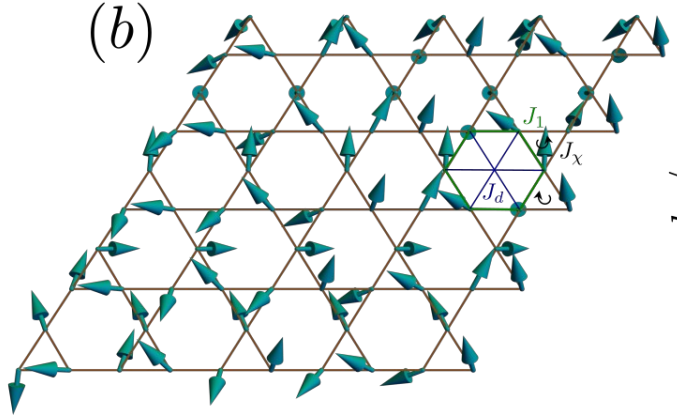
Benchmark: (A)FM chains weakly coupled $(J_d >> J_\chi)$

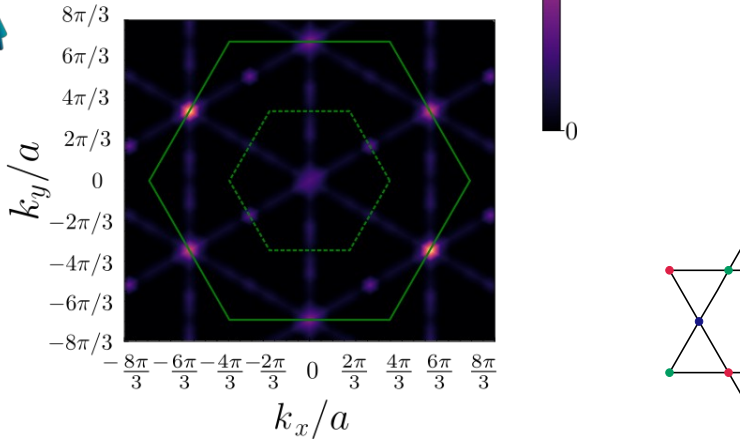
AFM Chains

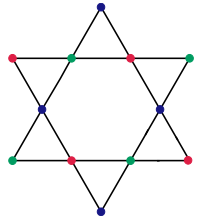




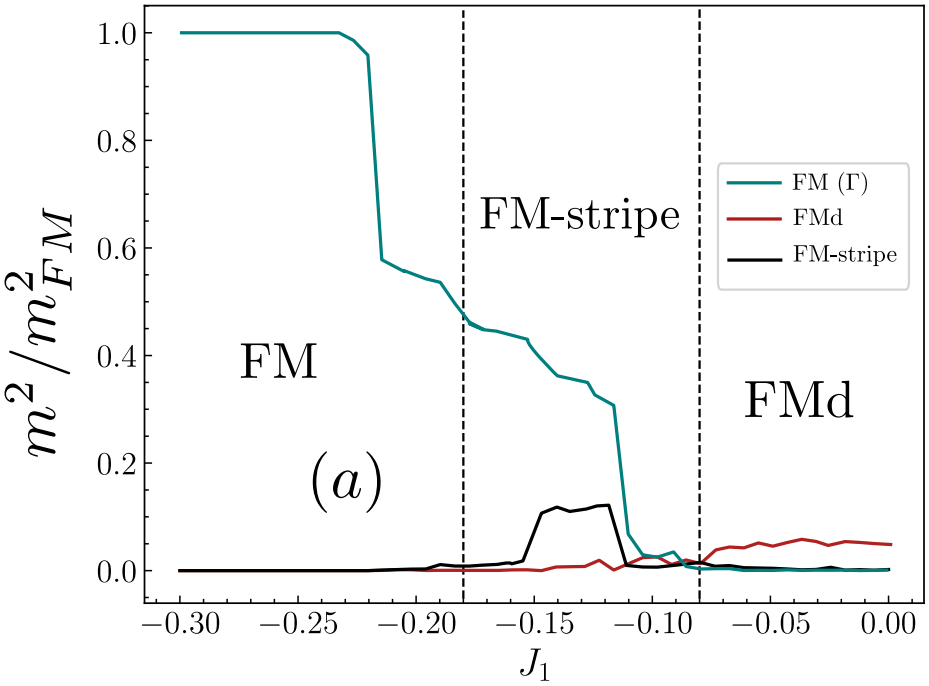
FM Chains



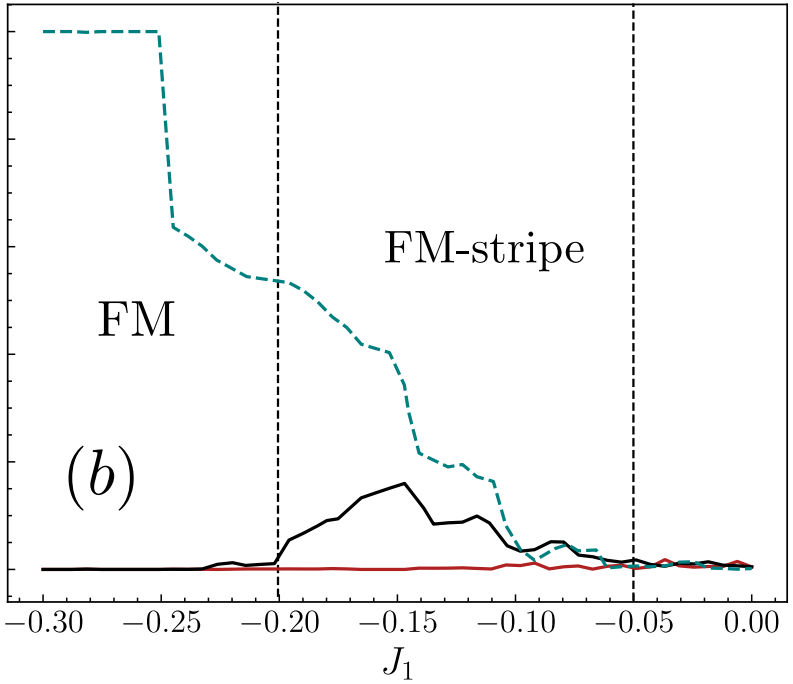




FM, FM-stripe, FMd phase transition



$(J_d = -0.20)$



$(J_d = -0.15)$

Shermann Morisson e Matrix Determinant Lemma

Algorithm 3 Variational Monte Carlo

1. Start with a random configuration in $\mathbb{A}_\alpha$, with $n_\uparrow = n_\downarrow$ in random positions. Store both inverse matrices and save a backup of each initial configuration.

2. Select a new configuration by randomly exchanging the position of two opposite spins. If the new spin is in the k-th column

$$\det \mathbb{A}_\beta = \left(1 + e_k^T \mathbb{A}_\alpha^{-1} u_k\right) \det \mathbb{A}_\alpha \tag{2.61}$$

and

$$\mathbb{A}_\beta^{-1} = \mathbb{A}_\alpha^{-1} - \frac{\mathbb{A}_\alpha^{-1} u_k e_k^T \mathbb{A}_\alpha^{-1}}{1 + e_k^T \mathbb{A}_\alpha^{-1} u_k}, \tag{2.62}$$

with $u_i = [\mathbb{A}_\beta]_{ik} - [\mathbb{A}_\alpha]_{ik}$.

3. Accept or reject the new configuration by the Metropolis rule

$$T(\alpha \to \beta) = \min\left[1, \frac{p(\beta)}{p(\alpha)}\right] = \begin{cases} \frac{p(\beta)}{p(\alpha)} & , \frac{p(\beta)}{p(\alpha)} < 1 \\ 1 & , \frac{p(\beta)}{p(\alpha)} \geq 1 \end{cases} . \tag{2.63}$$

where $\frac{p(\beta)}{p(\alpha)} = \frac{|\langle\beta|\psi\rangle|^2}{|\langle\alpha|\psi\rangle|^2} = \left|\frac{\det \mathbb{A}_\beta}{\det \mathbb{A}_\alpha}\right|^2 = \left(1 + u_k \mathbb{A}_\alpha^{-1} e_k^T\right)\left(1 + u_k \mathbb{A}_\alpha^{-1} e_k^T\right)^*$. This equation only states a minimization of energy. To properly introduce a Metropolis Algorithm one should sort a random number $r \in [0, 1]$: if $r < p(\beta)/p(\alpha)$ the new configuration can still be accepted. Otherwise, if $r \geq p(\beta)/p(\alpha)$, the old configuration is maintained.

4. Go back to step 2 and repeat the algorithm for the desired MC sweeps (one MC sweep is equivalent to N spin exchange attempts).

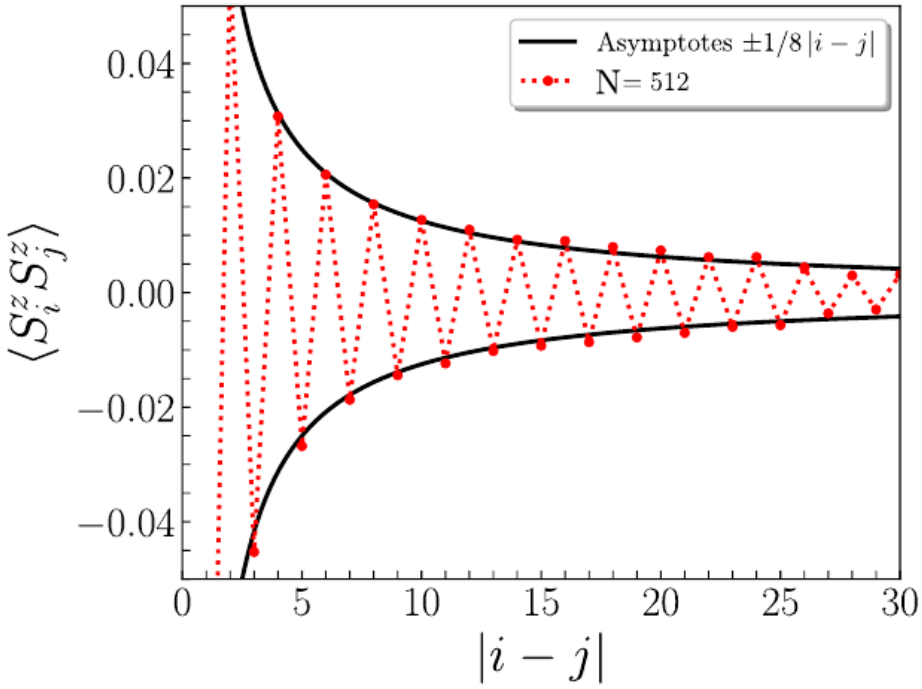
Haldane-Shastry / AFM Heisenberg Chain

N	E/JN $_{(NL)}$	E/JN $_{(L)}$
32	−0.44303 (5)	−0.4435 (4)
64	−0.44232 (5)	−0.4418 (4)
96	−0.44219 (5)	−0.4423 (3)
128	−0.44213 (5)	−0.4419 (3)
256	−0.44212 (5)	−0.4405 (3)
512	−0.44211 (4)	−0.4410 (3)

Source: By the author.

N	E^0_{HS}/JN	E_0/JN
32	−0.413 241 492	−0.413 241 492 (3)
64	−0.411 735 510	−0.411 735 510 (2)
96	−0.411 456 625	−0.411 456 625 (2)
128	−0.411 359 015	−0.411 359 015 (2)
256	−0.411 264 891	−0.411 264 891 (2)
512	−0.411 241 360	−0.411 241 358 (2)

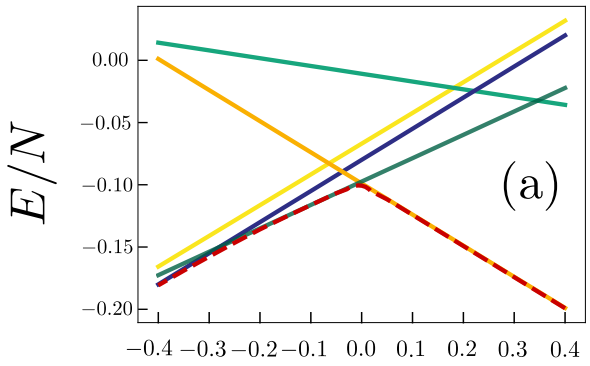
Source: By the author.



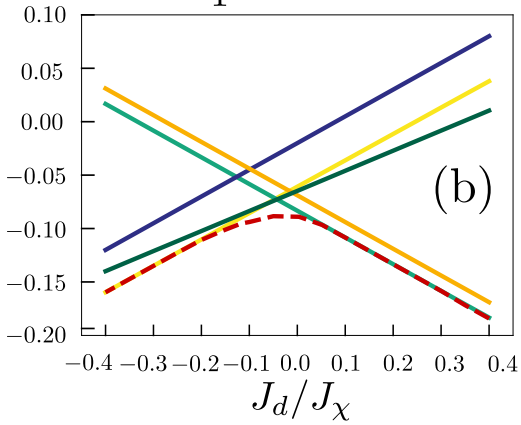
$$\mathcal{H}_{HS} = \sum_{p<q} \frac{J\pi^2 \boldsymbol{S}_p \cdot \boldsymbol{S}_q}{N^2 \sin^2 \frac{\pi}{N}(p-q)} = \frac{1}{2}\sum_{p=1}^{N}\sum_{q=1}^{N-1} \frac{J\pi^2 \boldsymbol{S}_p \cdot \boldsymbol{S}_q}{N^2 \sin^2 \frac{\pi}{N}(p-q)}$$

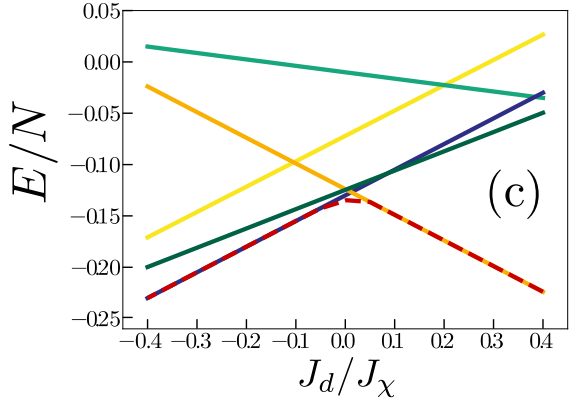
Classical phase diagram for the $\mathrm{J}_1 - J_d - J_\chi$ model

$\mathrm{J}_1 = -0.16$



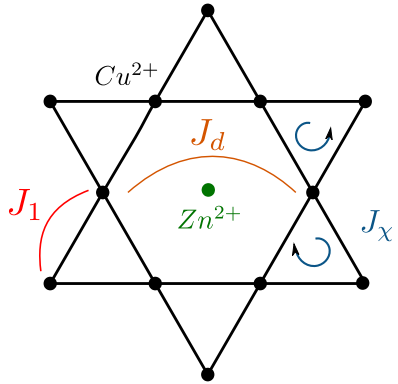
$\mathrm{J}_1 = -0.04$





$\mathrm{J}_1 = -0.26$

FMd
AFMd
FM
cuboc-2
FM-stripe
GD minimization



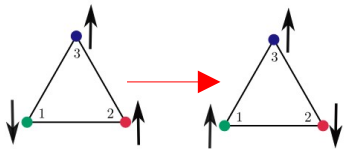
$$\mathrm{H} = \mathrm{H}_0 + H_\chi$$

$$H_0 = J_{ij} \sum_{\langle i,j \rangle} \mathbf{S}(\mathbf{r}_i) \cdot \mathbf{S}(\mathbf{r}_j)$$

$$H_\chi = J_\chi \sum_{ijk \in \triangle\triangledown} \mathbf{S}(\mathbf{r}_i) \cdot [\mathbf{S}(\mathbf{r}_j) \times \mathbf{S}(\mathbf{r}_k)]$$

$\mathrm{J}_1 < 0$ (FM), $J_d > 0$ (AFM) e $J_\chi > 0$

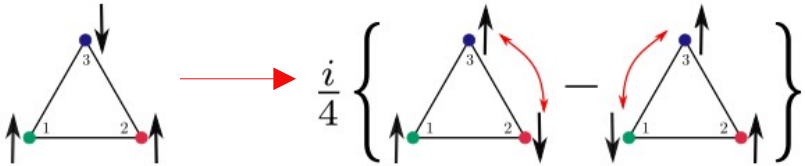
Parton Construction and VMC



$$H = \frac{1}{2}\sum_{\langle i,j\rangle} J_1 \left\{ \frac{1}{2}\left(f^\dagger_{i\uparrow} f_{i\downarrow} f^\dagger_{j\downarrow} f_{j\uparrow} + f^\dagger_{i\downarrow} f_{i\uparrow} f^\dagger_{j\uparrow} f_{j\downarrow} \right) + \left[\frac{1}{4} - \frac{1}{2}\left(f^\dagger_{i\uparrow} f_{i\uparrow} + f^\dagger_{j\uparrow} f_{j\uparrow} \right) + n_{i\uparrow} n_{j\uparrow} \right] \right\} +$$

$$+ \frac{1}{2}\sum_{i,j\in\hexagon} J_d \left\{ \frac{1}{2}\left(f^\dagger_{i\uparrow} f_{i\downarrow} f^\dagger_{j\downarrow} f_{j\uparrow} + f^\dagger_{i\downarrow} f_{i\uparrow} f^\dagger_{j\uparrow} f_{j\downarrow} \right) + \left[\frac{1}{4} - \frac{1}{2}\left(f^\dagger_{i\uparrow} f_{i\uparrow} + f^\dagger_{j\uparrow} f_{j\uparrow} \right) + n_{i\uparrow} n_{j\uparrow} \right] \right\} +$$

$$+ J_\chi \frac{i}{4} \left(\sum_{i\neq j\neq k\in\triangle}^{3} \varepsilon_{ijk} f^\dagger_{i\uparrow} f_{j\uparrow} f^\dagger_{j\downarrow} f_{i\downarrow} \left(n_{k\downarrow} - n_{k\uparrow} \right) - \sum_{i\neq j\neq k\in\triangledown}^{3} \varepsilon_{ijk} f^\dagger_{i\uparrow} f_{j\uparrow} f^\dagger_{j\downarrow} f_{i\downarrow} \left(n_{k\downarrow} - n_{k\uparrow} \right) \right)$$



$$\frac{i}{4}\left\{ f^\dagger_{3\uparrow} f_{2\uparrow} f^\dagger_{2\downarrow} f_{3\downarrow} n_{1\uparrow} + - f^\dagger_{3\uparrow} f_{1\uparrow} f^\dagger_{1\downarrow} f_{3\downarrow} n_{2\uparrow} \right\}$$

Parton Construction and Gutzwiller Projection

$$\chi_{i,j} \rightarrow \bar{\chi}_{i,j} e^{-ia_{i,j}}$$

↓

MFT ansatz

$$f_{i,\sigma} \rightarrow e^{i\theta_i} f_{i,\sigma}$$
$$a_{i,j} \rightarrow a_{i,j} + \theta_i - \theta_j$$

- Spinons coupled through a U(1) gauge field

Table 1 – Some examples of quantum spin liquids with their respective mean field theories, and possible low energy physics and excitations. For more examples see SAVARY *et al*[7] and ZHOU *et al.*[8]

	Mean Field Theory	Low energy & excitations
Superconductor	$\mathcal{H}_{MFT} = \sum_{ij} \left(f_i^\dagger f_j + \Delta_{ij} f_i^\dagger f_j^\dagger + \text{h.c.} \right)$	Topological QSL. Z_2 QSL Anyons, RVB
Semiconductor	$\mathcal{H}_{MFT} = \sum_i \varepsilon_i f_i^\dagger f_i + + \sum_{ij} \left(t_{ij} e^{ia_{ij}} f_i^\dagger f_j + \text{h.c.} \right)$	U(1) QSL Gapless photons
Semi-metal	$\mathcal{H}_{MFT} = \sum_{ij} \left(\tilde{t}_{ij} f_i^\dagger f_j + \text{h.c.} \right)$	U(1) QSL. Algebraic QSLs Dirac Fermions
Metal	$\mathcal{H}_{MFT} = \sum_{ij} \left(t_{ij} f_i^\dagger f_j + \text{h.c.} \right)$	U(1) QSL. Spinon Fermi Surface

Source: Adapted from SAVARY *et al.*[7]

Wen, Physical Review B 65.16 (2002): 165113.

Lucile Savary and Leon Balents (2017) Rep. Prog. Phys. 80 016502